

Toward Stochastic Explanation of Neutrally Stable Delayed Feedback Model of Human Balance Control

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Abstract

A stochastic explanation is provided to investigate how human subjects maximize robustness of their balance control while exhibiting on-off intermittent behavior. To this end, the human balance control is modeled by an inverted pendulum with random delayed state feedback. Stochastic analysis based on Lyapunov exponents demonstrates that the on-off intermittency can arise under a neutrally stable condition. Furthermore, the frequency response of statistical moments is derived to show that the neutrally stable condition can be caused by a trade-off between maximal robustness and minimal phase-shift from the disturbance to the second moments.

1 Introduction

One of the most marvelous features of human balance control is presence of on-off intermittency in balancing errors [1, 2]. In general, on-off intermittent behavior can arise in the system being nearly neutrally stable [3]. It follows that the human balance control seems to be tuned to be neutrally stable. Such a stability design is hardly obtainable from common approaches in control engineering because sufficient stability margins must be designed to achieve stable transient responses. The question arises as to what kind of performance the human subjects prefer to optimize rather than the asymptotic stability.

Presence of randomness in the human balance control have been explored in the studies on human sway control during quiet standing. It has been investigated that the human body during quiet standing continually moves about in a random fashion [4], and that the fluctuation-dissipation theorem can be applied into the human postural control system [5]. Furthermore, it has also been reported that input noise can be used to improve the human balance control [6] based on the mechanism of stochastic resonance [7] that is one of the most typical example of the noise-induced order [8].

Recently, researchers have recognized an additional feature of human balance control, on-off intermittency. It has already been shown that the on-off intermittency arising in the human balance control can be modeled precisely by an inverted pendulum with a randomly fluctuated time-delayed feedback controller [1], and that the

statistical property of the human stick balancing can be characterized as a special type of random walk, referred to as a Lévy flight, and statistically proved that the Lévy flight is deeply connected with learning process of human subjects to improve their balance control [2].

As mentioned in the opening paragraph, since the on-off intermittent behavior arises in the system nearly neutrally stable [3], the on-off intermittency generated by human, as reported in the literature [1, 2], implies that the human balance control is tuned near the minimally stable condition.

In this paper, we investigate the open problem of how the human balance control prefers the minimal stability. To this end, we focus on frequency responses of the model of human balance control [1]. A similar viewpoint can be found in the literature [9] based on direct numerical simulations. In contrast, the primary approach used in this study is based on the theory of stochastic processes. Effectiveness of such a stochastic approach have already been demonstrated in our previous studies such as the stability analysis of noise-induced synchronization [10, 11] and coupled human balancing [12, 13].

In practice, we derive a system of stochastic differential equations (SDE) representing the random delayed inverted pendulum model [1]. This SDE enables us to calculate Lyapunov exponents [14] evaluating the minimal stability of sample paths of balancing errors analytically. We also derive the moment equations [15] from the SDE to obtain the frequency response of statistical moments of the balancing errors. Based on these results, we will provide a stochastic explanation that the minimally stable condition seems to be caused by a trade-off between maximal robustness and minimal phase-shift from the disturbance to the second moments.

2 Analytical Model

2.1 Inverted pendulum

The equation of motion of an inverted pendulum whose pivot point is mounted on a cart is given by,

$$\begin{cases} (m_1 + m_2)\ddot{x} + (m_2 l \cos \theta)\ddot{\theta} - m_2 l \dot{\theta}^2 \sin \theta + c_x \dot{x} = F(t), \\ (m_2 l \cos \theta)\ddot{x} + (m_2 l^2)\ddot{\theta} - m_2 l g \sin \theta + c\dot{\theta} = 0. \end{cases} \quad (1)$$

where m_1, m_2 is a mass of the cart and the pendulum, l is a length of the rod considered massless, θ is the slant angle of pendulum, x is a horizontal displacement of the cart, and c and c_x are damping coefficients with respect to θ and x . For simplicity, assuming,

$$|\theta|, |\dot{\theta}| \ll 1, \quad m_1 = m_2 = m, \quad c_x = 0 \quad (2)$$

we obtain a linearized equation of motion with respect to the slant angle θ ,

$$\ddot{\theta} + \frac{2c}{ml^2}\dot{\theta} - \frac{2g}{l}\theta = -\frac{1}{ml}F(t) \quad (3)$$

in which the cart displacement x vanishes due to the linear approximation.

Using the natural frequency $\omega_n = \sqrt{2g/l}$, we perform a temporal scale transformation:

$$t \mapsto \omega_n^{-1}t. \quad (4)$$

Then, the equation of motion is reduced into the following form:

$$\ddot{\theta} + 2\zeta\dot{\theta} - \theta = f(t) \quad (5)$$

where $\zeta = c/(ml^2\omega_n)$ is a damping ratio and $f(t) = -F(t/\omega_n)/(ml\omega_n^2)$ is an external torque applied to the pendulum. The non-dimensional torque $f(t)$ is regarded as the combination,

$$f(t) = u(t) + v(t) \quad (6)$$

where $u(t)$ is a control input and $v(t)$ is an external disturbance.

2.2 Random delayed feedback

It has been reported that on-off intermittent behavior of the human balance control have precisely been modeled by a randomly fluctuated time-delayed feedback controller [1],

$$u(t) = -R_t\theta(t - \tau), \quad R_t = K + \sigma w_t \quad (7)$$

where R_t is a random gain with mean K and variance σ^2 , and w_t is a standard Gaussian white noise. In order to convert the delayed differential equation into an ordinary differential equation, we assume $\tau \ll 1$ and expand the delayed term in (7) as follows.

$$\begin{aligned} u(t) &\approx -R_t(\theta(t) - \dot{\theta}(t)\tau) \\ &= -K\theta + (K\tau)\dot{\theta} - w_t\sigma\theta - (\sigma\tau)\dot{\theta} \end{aligned} \quad (8)$$

Note that such linear approximation of delayed variables is often used in the engineering applications such as the machining chatter analysis [16].

Substituting (8) into (5) through (6), we obtain a state space expression of our model in the following form:

$$\begin{aligned} \dot{\mathbf{x}} &= A\mathbf{x} + \mathbf{b}v(t) + \sigma(D\mathbf{x})w_t, \quad \mathbf{x} = (\theta, \dot{\theta})^T, \\ A &= \begin{bmatrix} 0 & 1 \\ 1 - K & K\tau - 2\zeta \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ -1 & \tau \end{bmatrix}. \end{aligned} \quad (9)$$

3 Stability of Sample Paths

3.1 Standard form

In what follows, we denote the eigenvalues of the matrix A in (9) as

$$\lambda_{\pm} = \gamma \pm \sqrt{H} \quad (10)$$

where

$$\gamma = \frac{1}{2}(K\tau - 2\zeta), \quad H = \frac{1}{4}((K\tau - 2\zeta)^2 + 4(1 - K)).$$

In case of A having a pair of complex eigenvalues, the system matrices of the linear system (9) can be reduced into the following form:

$$T_C = \begin{bmatrix} 1 & 0 \\ \gamma & -\sqrt{-H} \end{bmatrix}, \quad (11)$$

$$A_C = T_C^{-1}AT_C = \begin{bmatrix} \gamma & -\sqrt{-H} \\ \sqrt{-H} & \gamma \end{bmatrix}, \quad (12)$$

$$\mathbf{b}_C = T_C^{-1}\mathbf{b} = \frac{1}{\sqrt{-H}} \begin{bmatrix} 0 \\ -1 \end{bmatrix}, \quad (13)$$

$$D_C = T_C^{-1}DT = \begin{bmatrix} 0 & 0 \\ (1 - \gamma\tau)/\sqrt{-H} & \tau \end{bmatrix}. \quad (14)$$

In case of A having two distinct real eigenvalues, the matrices are given in the following form:

$$T_R = \begin{bmatrix} 1 & 1 \\ \lambda_+ & \lambda_- \end{bmatrix}, \quad (15)$$

$$A_R = T_R^{-1}AT_R = \begin{bmatrix} \lambda_+ & 0 \\ 0 & \lambda_- \end{bmatrix}, \quad (16)$$

$$\mathbf{b}_R = T_R^{-1}\mathbf{b} = \frac{1}{2\sqrt{H}} \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad (17)$$

$$D_R = T_R^{-1}DT_R = \frac{1}{2\sqrt{H}} \begin{bmatrix} \lambda_+\tau - 1 & \lambda_-\tau - 1 \\ 1 - \lambda_+\tau & 1 - \lambda_-\tau \end{bmatrix}. \quad (18)$$

3.2 Lyapunov exponents

The random ordinary differential equation (ODE) in (9) can be rewritten as a stochastic differential equation (SDE) of the state $\mathbf{x} = (\theta, \dot{\theta})^T$ in the Stratonovich form:

$$d\mathbf{x} = (A\mathbf{x} + \mathbf{b}v(t))dt + (\sigma D\mathbf{x}) \circ dW_t \quad (19)$$

where W_t is a standard Brownian motion. By assuming $v(t) = 0$ and $\sigma \ll 1$, the Lyapunov exponent can be calculated in the following way [14].

In case of A having a pair of complex eigenvalues, $D = D_C$, Lyapunov exponent is given by

$$\begin{aligned} \lambda^\sigma &= \gamma + \frac{\sigma^2}{8}g_1 + o(\sigma^2), \\ g_1 &= (D_{12} + D_{21})^2 + (D_{22} - D_{11})^2. \end{aligned} \quad (20)$$

In case of A having two distinct real eigenvalues, $D = D_R$, Lyapunov exponent is given by

$$\lambda^\sigma = \lambda_+ + \frac{\sigma^2}{2}g_1 + o(\sigma^2), \quad g_1 = D_{12}D_{21} \quad (21)$$

where D_{ij} is (i, j) -th element of the matrix D .

4 Frequency Response of Moments

In order to utilize the Itô formula, the Stratonovich type equation (19) is converted into that of Itô type in the following form:

$$d\mathbf{x} = (A + \Delta A)\mathbf{x} + \mathbf{b}v(t) dt + (\sigma D\mathbf{x})dW_t \quad (22)$$

where $\Delta A = (\sigma D)^2/2$ is a drift correction term. Then, the statistical moments of (22) can be derived by the following way [15]. That is, the ensemble average $\langle h(\mathbf{x}) \rangle$ of a scalar function $h(\mathbf{x})$ s.t. $h(\mathbf{0}) = 0$ satisfies,

$$\frac{d\langle h(\mathbf{x}) \rangle}{dt} = \langle L(h(\mathbf{x})) \rangle \quad (23)$$

where

$$L(\cdot) = \left\{ (A + \Delta A)\mathbf{x} + \mathbf{b}v(t) \right\}^T \frac{\partial(\cdot)}{\partial \mathbf{x}} + \frac{\sigma^2}{2} \text{tr} \left\{ (D\mathbf{x})^T \frac{\partial}{\partial \mathbf{x}} \left(\frac{\partial(\cdot)}{\partial \mathbf{x}} \right)^T (D\mathbf{x}) \right\}. \quad (24)$$

is a generating operator.

4.1 Moment differential equations

Substituting $h(\mathbf{x}) = x_1, x_2, x_1^2, x_1x_2, x_2^2$ into (23), we obtain moment differential equations (MDE) in the following form:

$$\begin{aligned} \dot{m}_1 &= m_2, \\ \dot{m}_2 &= -k m_1 - c m_2 + v(t), \\ \dot{m}_{11} &= 2m_{12}, \\ \dot{m}_{12} &= -k m_{11} - c m_{12} + m_1 v(t), \\ \dot{m}_{22} &= \sigma^2 m_{11} + p m_{12} + q m_{22} + 2v(t) m_2, \end{aligned} \quad (25)$$

where

$$\begin{aligned} k &= K - 1 + \frac{1}{2}\sigma^2\tau, \quad c = 2\zeta - K\tau - \frac{1}{2}\sigma^2\tau^2, \\ p &= 2k - 2\sigma^2\tau, \quad q = -2c + \sigma^2\tau^2 \end{aligned} \quad (26)$$

and $m_i = \langle x_i \rangle$ and $m_{ij} = \langle x_i x_j \rangle$ ($i, j = 1, 2$) are ensemble averages of the state variables. Rewriting the second moments in (25) into the variance around the mean values, i.e., $s_{ij} = \langle (x_i - m_i)(x_j - m_j) \rangle$ ($i, j = 1, 2$), we obtain,

$$\begin{aligned} \dot{m}_1 &= m_2, \\ \dot{m}_2 &= -k m_1 - c m_2 + v(t), \\ \dot{s}_{11} &= 2s_{12}, \\ \dot{s}_{12} &= -k s_{11} - c s_{12} + s_{22}, \\ \dot{s}_{22} &= \sigma^2 s_{11} + p s_{12} + q s_{22} + \sigma^2(m_1 - m_2\tau)^2. \end{aligned} \quad (27)$$

Finally, taking the subspaces $\mathbf{m} = (m_1, m_2)^T$, $\mathbf{s} = (s_{11}, s_{12}, s_{22})^T$, we obtain the following form:

$$\dot{\mathbf{m}} = A_m \mathbf{m} + \mathbf{e}_2 v(t), \quad (28)$$

$$\dot{\mathbf{s}} = A_s \mathbf{s} + \mathbf{e}_3 Q(m_1, m_2), \quad (29)$$

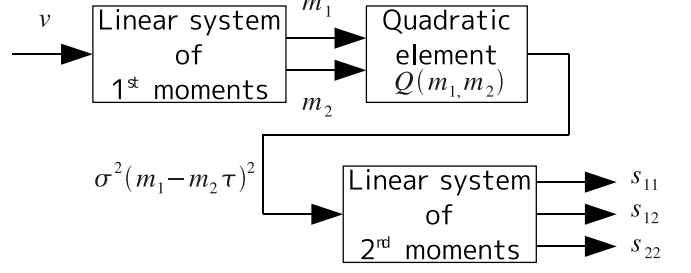


Fig. 1: Block diagram of MDE

where $\mathbf{e}_2 = (0, 1)^T$, $\mathbf{e}_3 = (0, 0, 1)^T$, and

$$A_m = \begin{bmatrix} 0 & 1 \\ -k & -c \end{bmatrix}, \quad A_s = \begin{bmatrix} 0 & 2 & 0 \\ -k & -c & 1 \\ \sigma^2 & p & q \end{bmatrix}, \quad (30)$$

$$Q(m_1, m_2) = \sigma^2(m_1 - m_2\tau)^2. \quad (31)$$

The most significant feature of this MDE is that the first moments \mathbf{m} in (28) can be solved by itself and that the second moments \mathbf{s} in (29) are subjected to the scalar-valued input $Q(m_1, m_2)$ only as shown in Fig.1. This makes it easier to solve this MDE.

4.2 Fundamental harmonic response

The structure of the MDE shown in Fig.1 allows us to derive the fundamental harmonic response of moments in a rigorous manner as follows.

Let us consider the harmonic disturbance,

$$v(t) = \cos \omega t \quad (32)$$

where the amplitude can be supposed to be unity without loss of generality.

4.2.1 The first moments

Based on the transfer matrix from $v(t)$ to $\mathbf{m}(t)$:

$$\mathbf{G}_m(a) = \begin{bmatrix} G_m^1(a) \\ G_m^2(a) \end{bmatrix} = (aI - A_m)^{-1} \mathbf{e}_2, \quad (33)$$

we obtain the fundamental harmonic response of the second order linear system (28) simply by

$$\begin{aligned} \mathbf{R}_m(\omega) &= \begin{bmatrix} R_m^1(\omega) \\ R_m^2(\omega) \end{bmatrix} = |\mathbf{G}_m^1(j\omega)| \\ &= \frac{1}{\sqrt{(k - \omega^2)^2 + c^2\omega^2}} \begin{bmatrix} 1 \\ \omega \end{bmatrix}, \end{aligned} \quad (34)$$

$$\begin{aligned} \phi_m(\omega) &= \begin{bmatrix} \phi_m^1(\omega) \\ \phi_m^2(\omega) \end{bmatrix} = \angle \mathbf{G}_m^1(j\omega) \\ &= \begin{bmatrix} -\arctan \frac{c\omega}{k - \omega^2} \\ \arctan \frac{k - \omega^2}{c\omega} \end{bmatrix} \end{aligned} \quad (35)$$

where $j = \sqrt{-1}$. Therefore, the fundamental harmonic response of the first moment is obtained as,

$$m_i(t) = R_m^i \cos\{\omega t + \phi_m^i\} \quad (i = 1, 2). \quad (36)$$

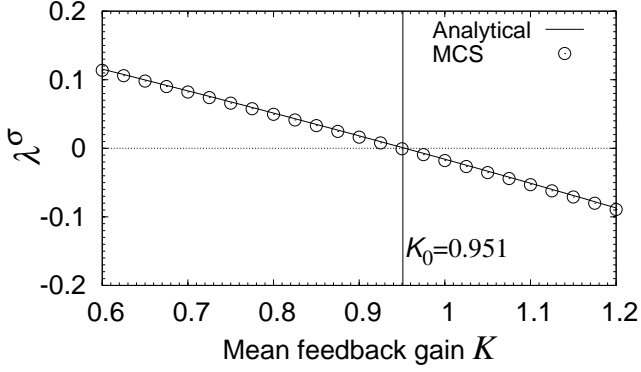


Fig. 2: Lyapunov exponent λ^σ as a function of the mean feedback gain K for $\zeta = 1.5$ and $\sigma = 0.5$.

4.2.2 The quadratic element

Substituting the first moments in (36) into the quadratic function $Q(m_1, m_2)$ in (31), we obtain,

$$Q(m_1, m_2) = R_Q(\omega) + R_Q(\omega) \cos\{2\omega t + \phi_Q(\omega)\}, \quad (37)$$

$$R_Q(\omega) = R_m^1(\omega)^2 \frac{\sigma^2}{2} \frac{1}{1 + (\omega\tau)^2}, \quad (38)$$

$$\phi_Q(\omega) = 2\phi_m^1(\omega) + \arctan \frac{2\omega\tau}{1 - (\omega\tau)^2}. \quad (39)$$

It appears that the output of quadratic element becomes a harmonic function having the doubled frequency 2ω and the drift term $R_Q(\omega)$.

4.2.3 The second moments

We now rewrite the equation of second moments in (29) around the static equilibrium \bar{s} satisfying

$$\mathbf{0} = A_s \bar{s} + e_3 R_Q(\omega). \quad (40)$$

Applying the transformation: $s = \bar{s} + s'$, we obtain the equation of second moments without the drift term:

$$\dot{s}' = A_s s' + e_3 w(t) \quad (41)$$

where

$$w(t) = R_Q(\omega) \cos\{2\omega t + \phi_Q(\omega)\} \quad (42)$$

is a harmonic function of a frequency 2ω .

Since the modified equation of the second moment obtained in (41) is a linear system subjected to the harmonic input, we can calculate its harmonic response, using the transfer matrix from $w(t)$ to $s'(t)$:

$$\mathbf{G}_s(a) = (aI - A_s)^{-1} e_3 \quad (43)$$

to obtain

$$\mathbf{R}'_s(\omega) = |\mathbf{G}_s(2j\omega)|, \quad \phi'_s(\omega) = \angle \mathbf{G}_m(2j\omega). \quad (44)$$

Therefore, the total contribution in amplitude and phase-shift from the disturbance $v(t)$ in (32) to the second moment s is given by

$$\mathbf{R}_s(\omega) = R_Q(\omega) \mathbf{R}'_s(\omega), \quad (45)$$

$$\phi_s(\omega) = \phi_Q(\omega)(1, 1, 1)^T + \phi'_s(\omega). \quad (46)$$

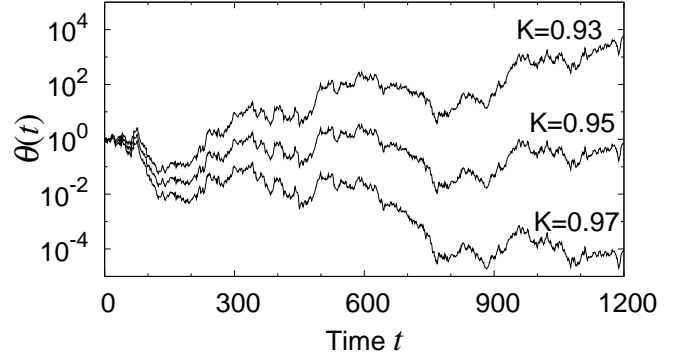


Fig. 3: Sample paths for $\zeta = 1.5$ and $\sigma = 0.5$.

In conclusion, we have obtained the fundamental harmonic response of moments in (34), (35), (45), and (46) without approximations, because of the particular structure of our MDE as shown in Fig.1.

4.3 Robust analysis of moments

Now we can define the H_∞ norm of moments from the harmonic response obtained above. This enables us to evaluate the robustness of moments against the disturbance $v(t)$. Since all of the transfer functions obtained above are vectors, the H_∞ norm of moments is simply defined by choosing the maximal component of the supremal amplitude vector. In this way, the H_∞ norm of moments is given by,

$$H_\infty^\alpha = \max_i \left\{ \sup_\omega \mathbf{R}_\alpha(\omega)_i \right\} \quad (\alpha = m, s) \quad (47)$$

where $[v]_i$ denotes the i th component of the vector v .

On the other hand, since all components of the phase-shifts, $\phi_m(\omega)$ and $\phi_s(\omega)$, are monotonically decreasing functions of ω whose infimal values are negative constants independent of the system parameters, i.e., $\inf_\omega \phi_m(\omega) = (-\pi, -\pi/2)^T$, $\inf_\omega \phi_s(\omega) = (-4.5\pi, -4\pi, -3.5\pi)^T$. This means that the dependency on the system parameters can not be evaluated by a scalar index. To avoid this problem, we evaluate the maximal phase-shift at ω taking the negative maximum of components as follows.

$$\phi_\infty^\alpha(\omega) = -\max_i \left\{ -\phi_\alpha(\omega)_i \right\} \quad (\alpha = m, s). \quad (48)$$

5 Numerical Results

5.1 On-off intermittency

Figure 2 shows Lyapunov exponent λ^σ of the model (19) as a function of the mean feedback gain K for $\tau = 0.04$, $\zeta = 1.5$ and $\sigma = 0.5$ where the solid line represents the analytical result from the formula (21), corresponding to the over-damping condition $\zeta = 1.5$, and the small circles represents the result from Monte Carlo simulation of the random ODE in (9).

It appears in Fig.2 that $\lambda^\sigma = \lambda^\sigma(K)$ is a monotonically decreasing function of K , having the zero point near $K =$

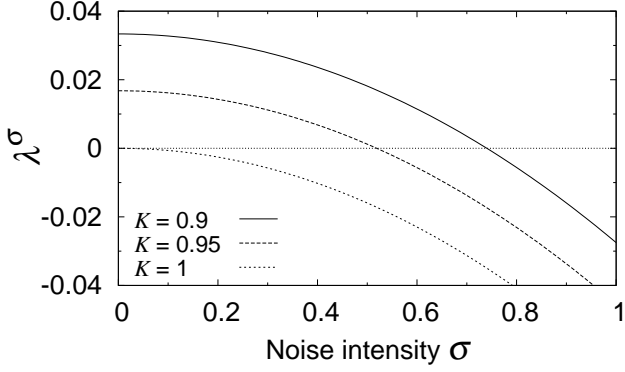


Fig. 4: Lyapunov exponent λ^σ as a function of the noise intensity σ for $\zeta = 1.5$.

$K_0 \approx 0.951$. This is a predictable result because the Lyapunov exponent λ^σ is a stochastic counter part of the real part of eigenvalues. Thus, the sufficiently large feedback gain K can make the real part of eigenvalues all negative in the deterministic limit $\sigma \rightarrow 0$.

Fig.3 shows sample paths near the zero point $K = K_0$ obtained from the random ODE (9) where a sample of the noise w_t is identical to each case. It appears that the state $\theta(t)$ diverges for the smaller gain $K = 0.93 < K_0$ and converges for the larger gain $K = 0.97 > K_0$. On the other hand, near the zero point $K = 0.95 \approx K_0$, the bounded state wandering around $|\theta(t)| \approx 10^0$ appears, which is the on-off intermittent behavior we will consider.

For reference, Fig.4 shows dependency of the Lyapunov exponent λ^σ upon the noise intensity σ . This result can be regarded as an example of the noise-induced order [8] because λ^σ is monotonically decreasing function of σ and is structurally stable with respect to the change of mean feedback gain $K = 0.9, 0.95, 1$.

5.2 Amplitude of moments

Fig.5 shows Lyapunov exponent λ^σ and H_∞ norms as functions of the mean feedback gain K for $\sigma = 0.5$ where H_∞ is calculated following the definition (47). It clearly appears that $H_\infty(K)$ is a concave function of K , whose peak is placed at $K = K_p$ larger than the zero point $K = K_0$.

This result allows us to provide a possible explanation of the human nature preferring the minimally stable mean feedback gain K_0 . It clearly appears in Fig.5 that the gain K_0 locally minimizes $H_\infty(K)$ under the constraint $\lambda^\sigma(K) < 0$ avoiding dynamic instabilities. Therefore, it seems that the gain K_0 preferred by human locally maximizes the robustness of the moments m, s . The same consideration can be done for the larger noise intensity $\sigma = 1$ as show in Fig.6.

However, this explanation is limited to local domain of K because the same robustness can be found globally at

$$K_1 = H_\infty^{-1}(H_\infty(K_0)) \quad (49)$$

as shown in Fig.5 and 6. Since the second typical gain K_1 provides the same extent of $H_\infty(K)$ or the robustness, the human could prefer K_1 without changing the

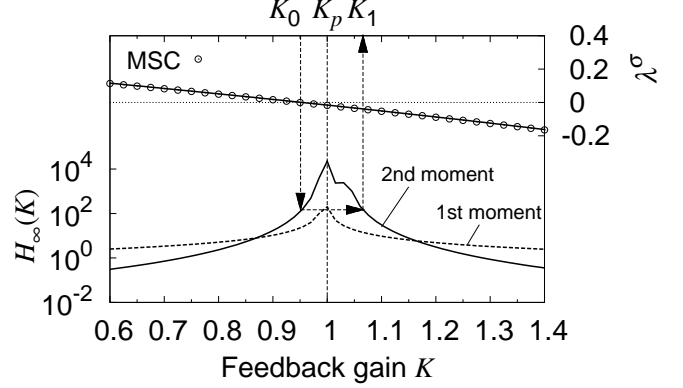


Fig. 5: Lyapunov exponent λ^σ and H_∞ norms as a function of the mean feedback gain K for $\sigma = 0.5$.

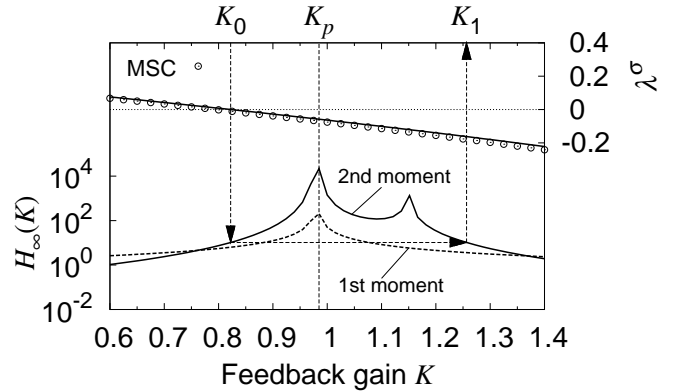


Fig. 6: Lyapunov exponent λ^σ and H_∞ norms as a function of the mean feedback gain K for $\sigma = 1$.

robustness. Furthermore, the second gain K_1 provides asymptotic stability stronger than K_0 ?

5.3 Phase-shift of moments

One explanation to answer the selection of the minimally stable gain K_0 is given by investigating the phase-shift of moments defined in (48) as follows.

Figure 7 shows the difference of maximal phase-shifts between at $K = K_0$ and K_1 for $\nu = 0.5$ as a function of the input frequency ω of the disturbance $v(t)$ in (32). It is clearly shown in Fig.7 that switching the gain from K_0 to K_1 results in significant increase of the maximal phase-shift of second moments.

This result implies that the minimally stable gain K_0 that seems to be preferred by human subjects produces the phase-shift significantly smaller than the more stable gain K_1 . The same conclusion can be obtained from the different extent of noise $\sigma = 1$ as shown in Fig.8.

6 Concluding Remarks

The above results clearly show that the minimally stable condition K_0 can be characterized as a special condition that minimizes the magnitude of the maximal am-

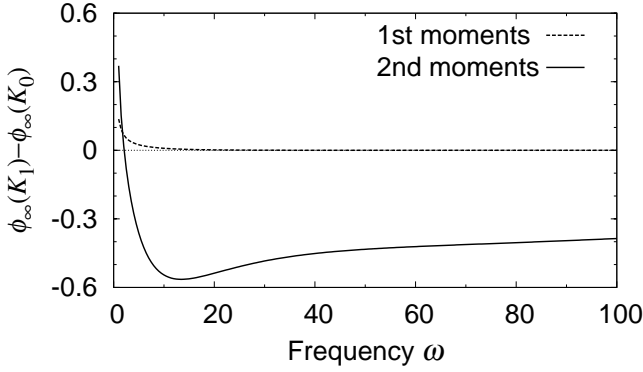


Fig. 7: Difference between the maximal phase-shifts at $K = K_0$ and K_1 for $\nu = 0.5$.

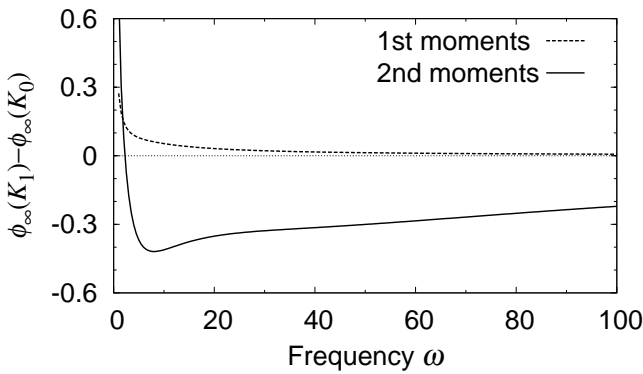


Fig. 8: Difference between the maximal phase-shifts at $K = K_0$ and K_1 for $\nu = 1$.

plitude H_∞^s and phase-shift ϕ_∞^s of second moments, subjected to the constraint $\lambda^\sigma < 0$ to avoid the dynamic instabilities. In conclusion, an stochastic explanation is obtained that the human subjects seem to prefer better robustness and faster cognition of predictability (smaller phase-shift of second moments) while accepting the minimally stable condition.

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